Application of the Push-Relabel Algorithm to Lignite Surface Mine Optimisation

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Open Pit Optimisation

- Open pit optimisation is a process commonly applied in mine planning of surface mines to produce optimum pit limits to use as a guide for pit design.
- It is also considered an efficient way to convert mineral resources to mineral reserves as it allows the enforcing of financial and technical constraints and parameters to the mine design process in an automated and mathematically robust way.
- It is commonly used even at the mineral resources estimation stage to limit the reported quantities inside a conceptual pit and raise the confidence in the mineral resources report.

Surface Coal Mine Design

- Surface coal and lignite mines have been commonly modelled in the past using a more two-dimensional approach, based on grid or triangulation models that did not allow the application of open pit optimisation algorithms, normally requiring a three-dimensional blocks model of the deposit.
- The financial aspects of coal deposits are also considered stable along the Z axis, in most cases where the deposit consists of a small number of coal horizons with standard qualities, leading to the conception that pit optimisation is an unnecessary effort.
- The lignite deposits in Greece, however, normally consist of multiple lignite layers with varying quality parameters in all three dimensions, making them ideal targets for computerised open pit optimisation.

Pit Optimisation Methods

- ***Floating Cone:*** introduced by Pana (1965) and developed at Kennecott Copper Corporation during the early 1960s as the first computerised attempt at pit optimization, based on a three-dimensional block model of the mineral deposit.
- ***Lerchs-Grossman:*** introduced by Lerchs and Grossman in 1965 based on two mathematical techniques – Graph Theory and Dynamic Programming.
- ***Maximum Flow and Push-Relabel:*** a decade later Picard proved that the pit optimisation problem could be solved with more efficient maximum flow algorithms, while in 1988 Goldberg and Tarjan published the first paper describing the Push-Relabel algorithm for solving the maximum flow problem.

Floating Cone Method

- It was developed at Kennecott Copper Corporation during the early 1960s and was the first computerised attempt at pit optimization, based on a three-dimensional block model of the mineral deposit.
- Final pit limits are developed by using a technique of a moving “cone” (or rather an inverted cone).
- The cone is moved around in the block model space from top to bottom generating a series of interlocking cone-shaped openings.
- The disadvantage of this approach is that it creates overlapping cones, and it is incapable of examining all combinations of adjacent blocks.
- For this reason, the algorithm fails to consistently give realistic results and tends to “mine” more tonnage for less value.

Lerchs-Grossman Method

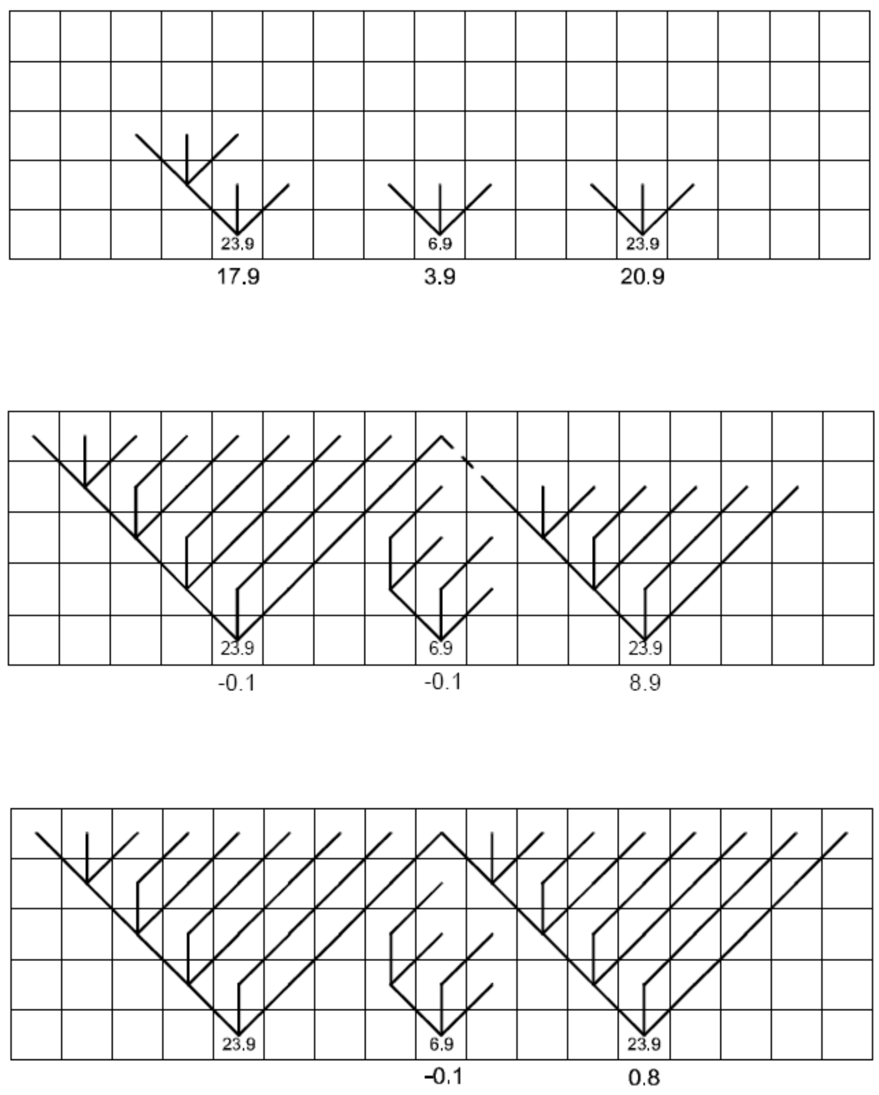
- Both implementations of the LG method produced optimum pit limits based on an undiscounted cash flow – an economic block model including both ore and waste.
- They determine which blocks should be mined to obtain the maximum value from the pit.
- LG requires a technical and a financial parameter:
  - ***Pit slopes***: these define the blocks that need to be removed before each block considered in the block model. They are used to generate "arcs" between blocks.
  - ***Block value***: refers to the economic value of each uncovered block. It will be negative for waste blocks and amount to all waste mining and hauling costs. Ore blocks will have values based on the mining, hauling, processing, selling and any other costs, and the revenue from the recovered ore.

Lerchs-Grossman Method

- Working from the lowest positive block(s) and using the block values and structure arcs, the method branches upwards between blocks forming a graph.
- Branches are flagged based on their total value.
- Positive branches are worth mining once uncovered.
- The scanning is repeated until no structure arc goes from a positive branch to a negative.
- Once this is complete, the complete graph defines the optimum pit.

Lerchs-Grossman Method

- In mathematical terms, LG finds the maximum closure of a weighted directed graph.
- The blocks represent the vertices of the graph, their values represent the weights, and the pit slopes represent the arcs.
- The algorithm itself has no “sense” of the nature of the optimisation problem – it works on a set of vertices and arcs.



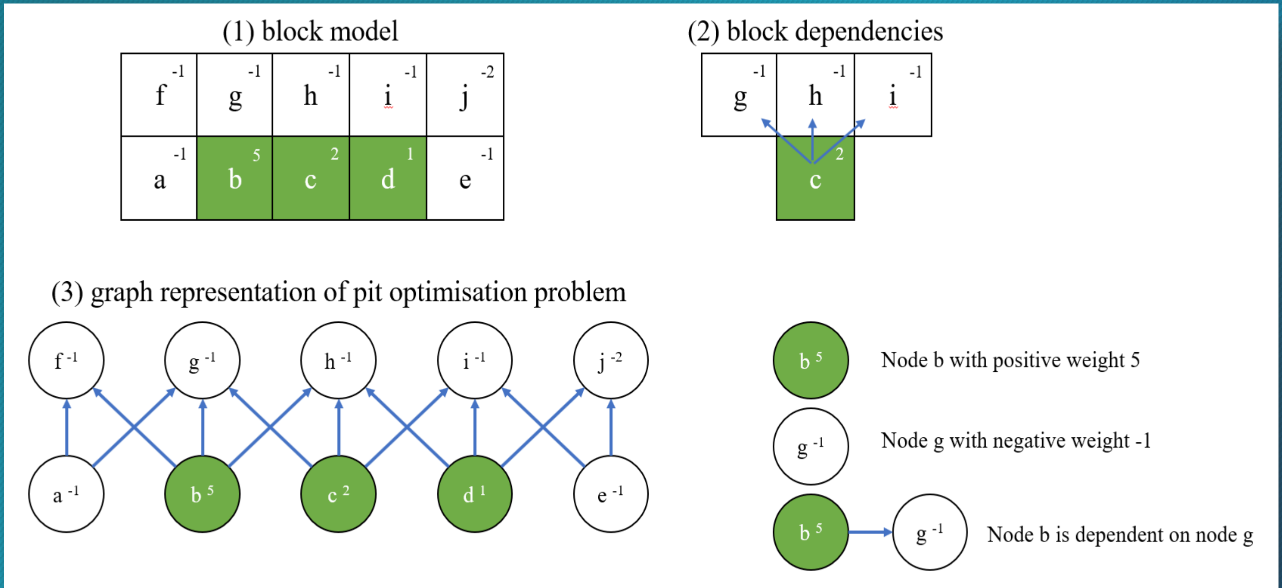
Maximum Flow Methods

- The maximum flow problem is a classical combinatorial problem that arises in a wide variety of applications.
- The basic methods for the maximum flow problem include the network simplex method of Dantzig, the augmenting path method of Ford and Fulkerson, the blocking flow method of Dinitz, and the push-relabel method of Goldberg and Tarjan.
- Prior to the push-relabel method, several studies have shown that Dinitz's algorithm is in practice superior to other methods.
- Several recent studies show that the push-relabel method is superior to Dinitz's method in practice.

Maximum Closure Problem

- The definition of a pit with valid slopes is termed a "closed set" or "closure".
- It consists of a set of nodes *V* that have no arcs initially.
- Based on the required pit slopes, a set of arcs *E* is defined representing the dependencies between blocks.
- A closed set of blocks is free to be removed and does not depend on the removal of other blocks.
- Finding an optimal pit is the process of finding a closure with maximum total value.

Graph Representation of the Pit Optimisation Problem

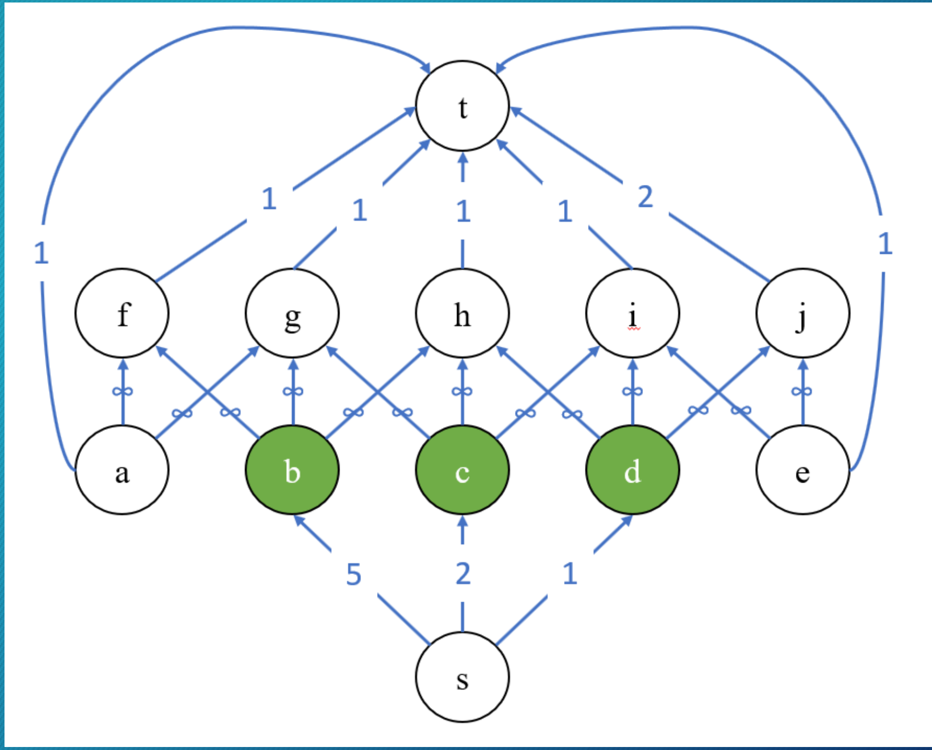


The optimal pit consists of block {b, c, f, g, h, i}, with a total value of 3.

Flow Graph of a Block Model

Defining a complete flow graph means that we need to make the following changes to the graph of the block model :

- Add two special (virtual) nodes: source *s* and sink node *t*.
- For all the existing arcs (blue), assign infinite capacities.
- Add links from source to all positive nodes, with the capacities equal to the weight of the nodes.
- Add links from negative nodes to sink, with the capacities equal to the absolute weight value of the nodes.
- Remove the weights on nodes.

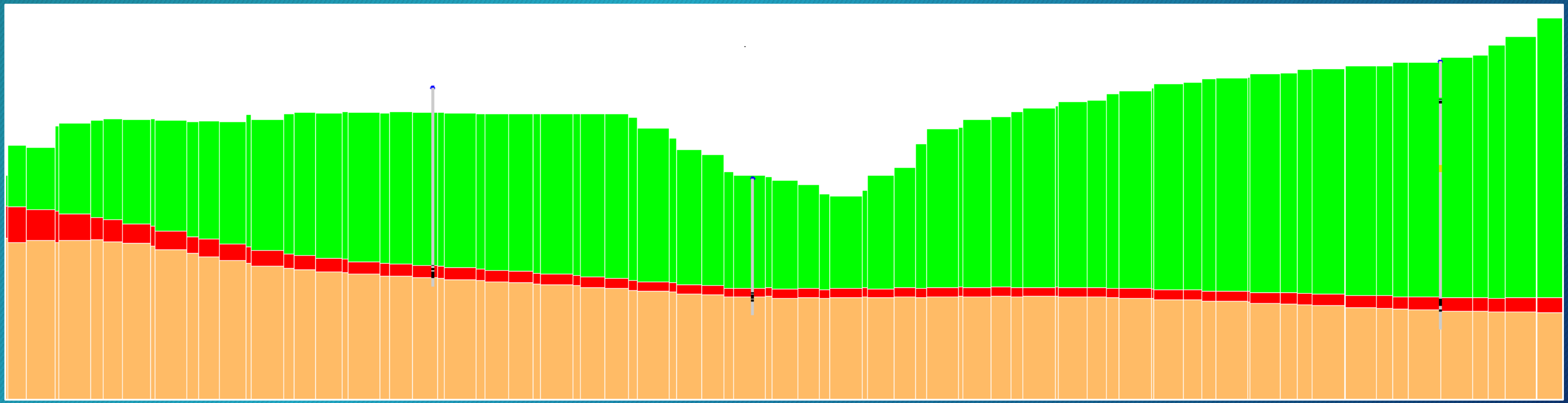


Push-Relabel Method

- The push-relabel method maintains a preflow f, initially set to zero an all arcs, and a distance labelling d.
- The $d(v)$ is initially set to the distance from node v to t in the graph.
- In its first stage, the push-relabel method repeatedly performs the update operations, ***push*** and ***relabel*** until there are no active nodes left.
- The update operations modify the preflow f and the labelling d.
- The second stage of the method converts f into a flow.

Case Study

- A lignite deposit from the area of West Macedonia in NW Greece was used in the study.
- It consists of a few lignite layers, and a simpler structure compared to other lignite deposits commonly found in the area.
- The roof, floor and qualities of the mineable lignite area of the deposit were modelled as grid surfaces using inverse distance interpolation.
- These grid models were used to generate a stratigraphic block model in Maptek Vulcan

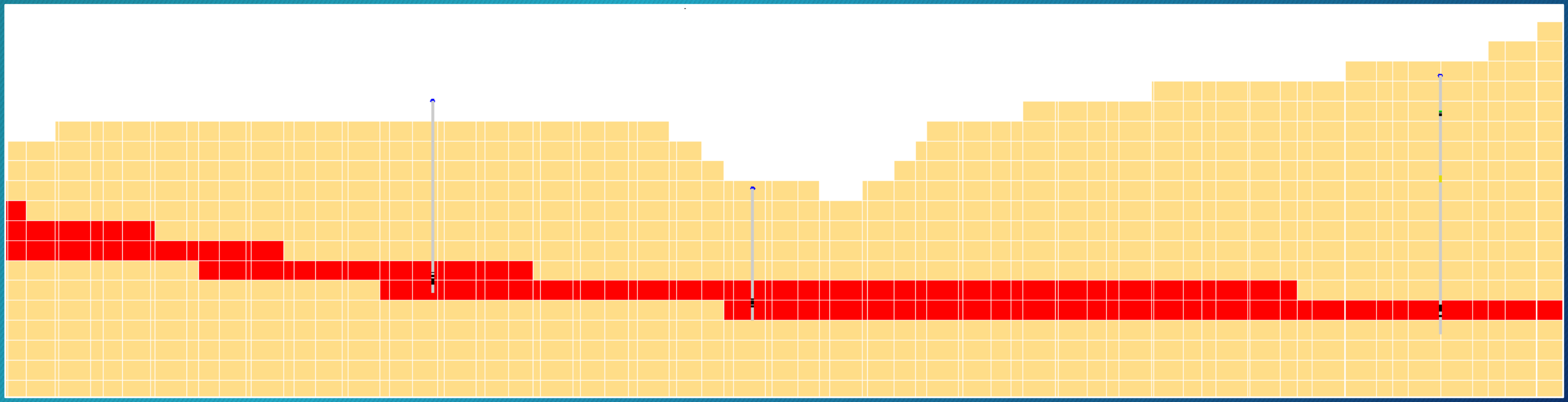


Financial Parameters of Optimisation

- Financial parameters included all mining and processing related costs, and revenue from selling of recovered lignite.
- The calculation of these parameters was based on the volume of each block, the thickness of mineable lignite and parting, the specific gravity for lignite and waste, and the type of each block (overburden, lignite deposit, underburden).
- the undiscounted cash flow of uncovered blocks, necessary as input for the pit optimisation process, was also calculated.
- The value was positive for lignite deposit blocks and negative for overburden and underburden blocks.

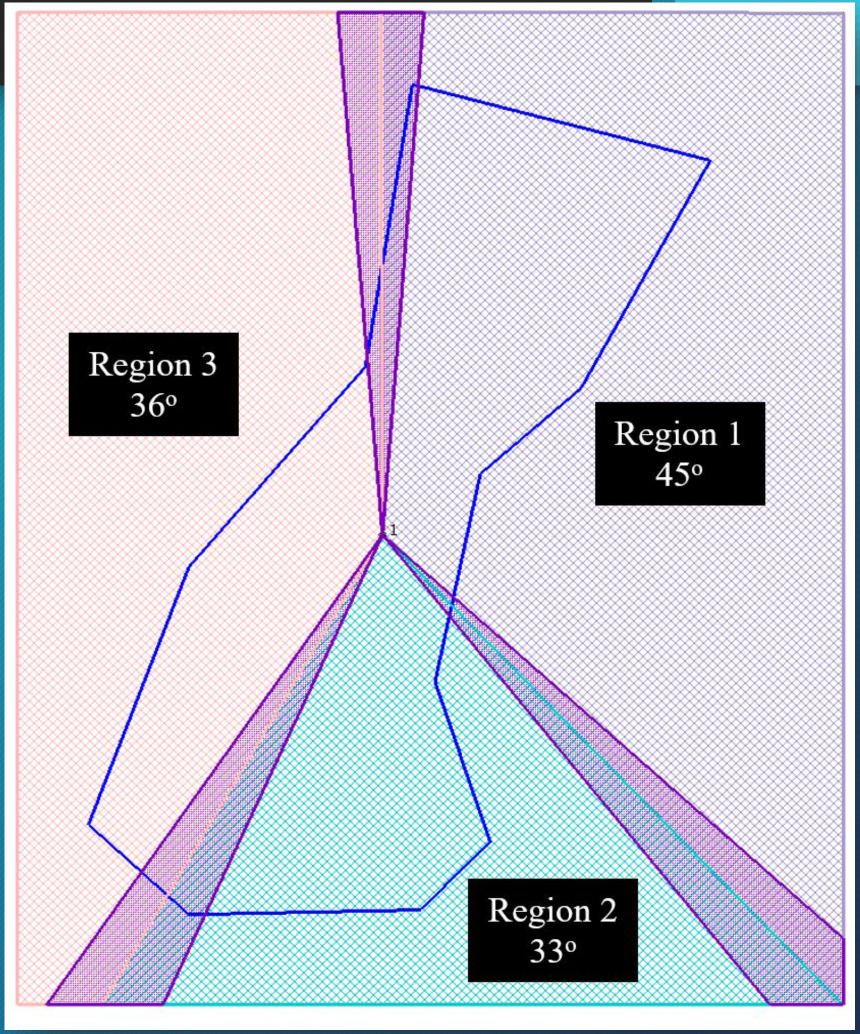
Block Regularisation

- All current methods of pit optimisation require a regular block model, i.e. a model with equally sized (regular) blocks.
- This meant that the stratigraphic block model that contained the calculated block values had to be regularised to a standard block size (10x10x8m).
- Only the block value variable was transferred to the regularised block model as it was the only parameter necessary as input to pit optimisation.
- This variable was calculated for each block using a sum of the intersecting stratigraphic model blocks' values weighted by their volume inside the regular block.



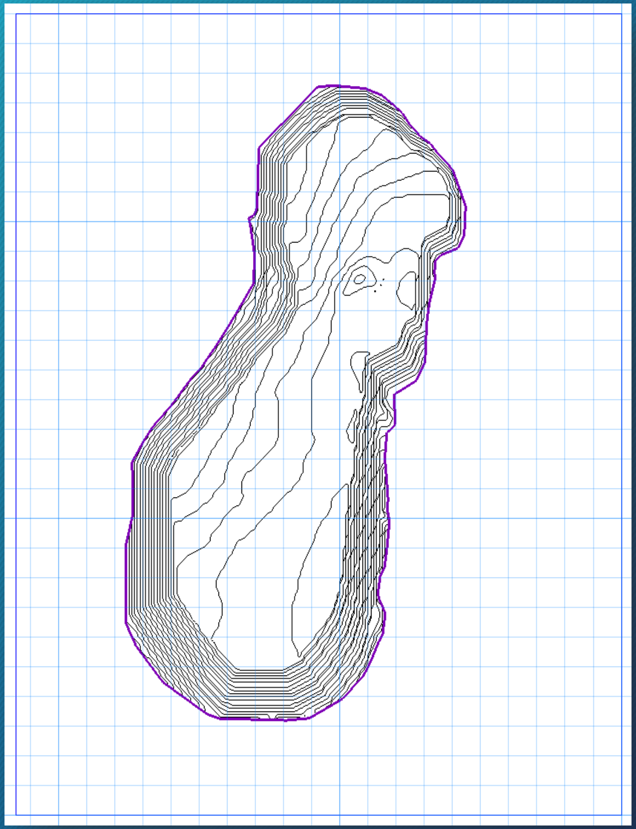
Technical Parameters of Optimisation

- The second piece of information required by the pit optimisation process is the required pit slopes.
- These were based on a conceptual geological model of the deposit area and information related to the stability of different types of rock.
- The area to be optimised was split into three slope regions based on azimuth.
- A 10º slope interpolation area was used to transit between slope regions.



LG vs Push-Relabel Comparison

- Two separate pit optimisation runs were set up using the same input information (block model and pit slope regions) – one for the LG method and one for Push-Relabel.
- The pit limits and pit value produced were 100% identical between the two methods.
- The two optimisation runs produced the same result numerically and geometrically to the last block.
- However, LG required **one hour and 45 minutes** to complete the optimisation while Push-Relabel required one minute and 33 seconds!



Conclusions

- Coal and lignite deposits were not so often approached and designed using pit optimisation.
- The case study presented shows that there is value in using pit optimisation for lignite deposits and that the current methods can provide a consistent and efficient way to limit the extents of lignite mines both horizontally and vertically.
- Speed improvements of the Push-Relabel method open up the opportunity to solve problems consisting of millions of blocks (such as large lignite mines) that were previously too large for the traditional LG method.

Thank you for your attention!