Use of Tetrahedral Modelling for Variography and Grade Estimation of a Structurally Deformed Phosphate Deposit

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ABSTRACT : The application of geological controls in orebody evaluation and modelling is critical. Stratigraphic orebodies require more accurate location in space and better seam boundary definition. This is achieved by either modelling boundary surfaces of the orebodies or their thickness at specific points in 3D space. Structurally deformed stratigraphic deposits pose a very special problem in variography and grade estimation – it is necessary to bring the locations of samples from the deformed space to the standard XYZ co-ordinate system. This is necessary in order to reconstruct the spatial distribution of grades at the time the deposit was formed and restore the relative positions of samples to their pre-deformed state. Earlier methods of unfolding deposits were based on various geometrical, mathematical or even manual techniques, while more recent methods were based on the use of an unfolded coordinate system for the transformation of every sample and estimation point. The alternative method presented in this paper, Tetrahedral Modelling, can be applied to deposits where mineralization is controlled by a pair of structural surfaces that can be modelled. In tetrahedral modelling the search ellipse is distorted to follow nominated structural surfaces leading to improved estimation accuracy. A case study is presented showing the benefits of this technique.

KEYWORDS: grade estimation, variography, unfolding, stratigraphic deposits, tetrahedral modelling.

1. Introduction

The dynamics of folding in mineral deposits have been extensively studied in several analyses and simulations. A number of methods for modelling deformations produced by various folding mechanisms have been developed. Earlier methods of unfolding deposits were based on various geometrical, mathematical or even manual techniques such as least squares, cylindrical unfolding or the fitting of splines (Royle 1979, Dagbert et al. 1983, Dowd 1986). Other more recent methods were based on the use of an unfolded coordinate system for the transformation of every sample and every estimation point for variography and grade estimation (Newton 1995). Tetrahedral modelling for variography and grade estimation was developed by Trevor Coulsen in 1995 (Maptek Pty Ltd) and implemented in Maptek's VULCAN 3D software package. Further improvements to the original algorithm were made by Peter Borovina (Maptek Pty Ltd) in 2002. Tetrahedral modelling is a method of adjusting the search ellipse used in variography and grade estimation to follow the geometrical structure of the deposit by forming a 3D tetrahedral model of the deposit volume. For this model to be generated, the structural surfaces of the deposit need to be modelled as surface triangulations.

2. Modelling of Structural Surfaces

Orebody outlines are usually interpreted by geologists from drillhole and development intersections on or near the plane of a cross section. Automated methods for geometrical modelling of stratigraphic horizons also exist but these are generally limited to cases where the horizons are fairly flat and consistent. Regardless of the method used, the geometrical model of the orebody needs to be generated and used to constrain the estimation process. Intersection points with a stratigraphic orebody's roof or floor can be used to create a surface approximated by a regular grid or triangulation model and an interpolation algorithm.

2.1. Surface Triangulations Using a Projection Plane

Surface triangulations are structures consisting of an irregular mesh of triangle facets, built from a three-dimensional spatial distribution of point data and/or strings. All facets in a surface triangulation are unique in their spatial extent with respect to the plane of projection, i.e. no overlaps are allowed in this plane. The formation of facet edges is controlled by joining individual data points to form strings. Facet edges are not allowed to intersect these strings (also referred to as breaklines). The standard Delaunay triangulation algorithm attempts to create equiangular triangle facets honouring all strings as breaklines. The algorithm works in a plane that can be nominated parallel to the structure that is to be modelled. The proper use of a projection plane reflecting the average trend of the data can remove overhangs and often resolve ambiguities caused by the sampling pattern.

2.2. Surface Triangulations in 3D

In cases where it is impossible to find a plane that can be used to effectively model the available data using the Delaunay algorithm, like in overturned folded or reverse faulted surfaces, a different algorithm needs to be used. The solid triangulation creation routine implemented in VULCAN can be used to create models of complex stratigraphic boundaries. This routine joins closed or open strings of points with triangle facets and is ideal for generating complex models from data in sections. The sections containing the point or string data are not required to be parallel to each other. In cases of extreme variation of the modelled structure shape from one section to the next, tie lines are used to guide the triangulation routine. In the example presented in this paper, tie lines were used to join fold noses and where there was a significant difference in the density of data between sections.

3. Creating a Tetrahedral Model

All volumetric geometries can be represented in 3D using a set of tetrahedra. Tetrahedral modelling uses the tetrahedron as the basic unit for representing volumetric geometry. A tetrahedral model is composed of indexed 3D tetrahedra, in contrast to the set of connected flat triangles forming a standard triangulation. In the computer graphics and finite element research areas there is a growing interest in the use of tetrahedra for modelling volumes in 3D. Several computer algorithms exist for the generation of tetrahedral models from open or closed surfaces represented by point or string data (Danovaro et al. 2003, Blandford et al. 2003, Jones et al. 2003).

In the example shown in this paper, upper and lower surface triangulations of a strata-bound phosphate deposit were used to create a solid 3D tetrahedral model. Each indexed tetrahedron is represented by four integers that index a list of vertex coordinates. During block model grade estimation, the search ellipse (Fig. 1) was based on the shape of the tetrahedral model around each block.

The main stage in the tetrahedral modelling process is the generation of the tetrahedral model from the roof and floor surface models. This process is based on the generation of tetrahedra from the triangle points contained in the triangulation models of the roof and floor surfaces. The points are joined together to form tetrahedral shapes alternating in direction. Each triangle in the original two surfaces becomes a face in some tetrahedron in the resulting tetrahedral model. In cases when this is not possible, points are inserted (e.g. the midpoint of a triangle) to ensure that the original triangles do appear in the result. Further to this, each tetrahedron must not have all of its points coming from only one of the input surfaces. This requires internally rearranging the tetrahedra and possibly adding further points. The line segments generated pass through block model cells with one end point touching the hanging surface and the other end point touching the floor surface. The quality and resolution of the produced tetrahedral model depends on the point density of the limiting surfaces, especially in the areas where folding or faulting is more severe.

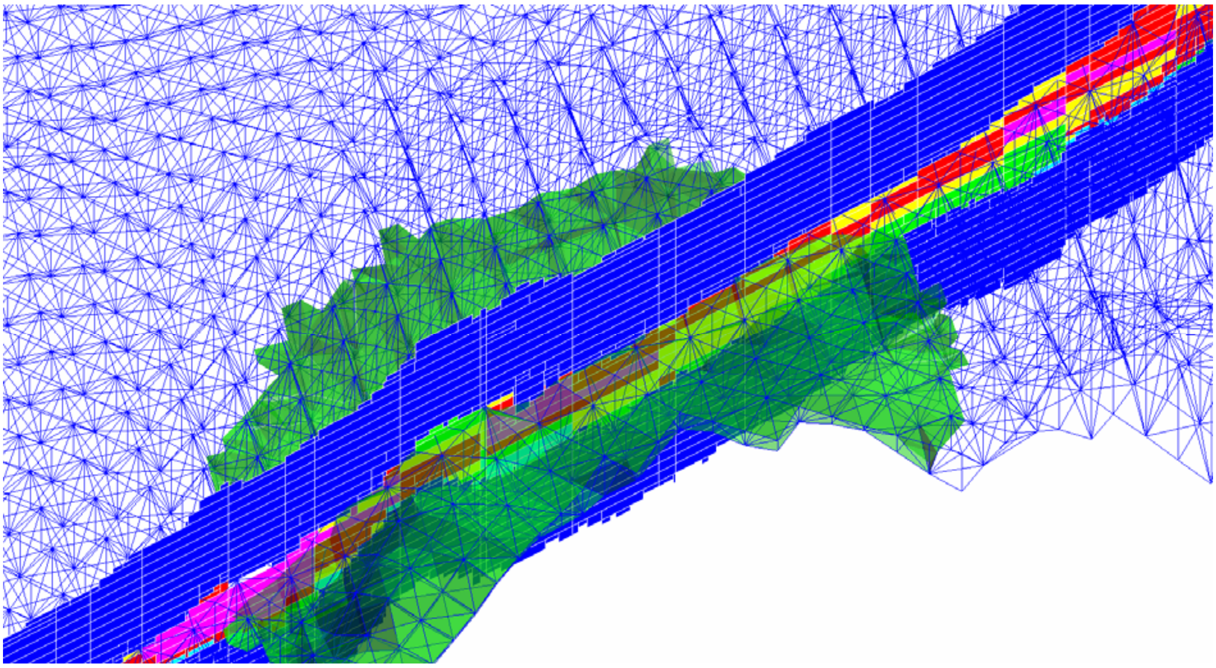

Fig. 1. Tetrahedral model constructed between roof and floor of phosphate seam, distorted search ellipse following tetrahedral model shape and extents, and block model slice showing estimation based on distorted search ellipse.

A line of minimum distance (true thickness) is calculated for each block cell. The line of minimum distance is then used to define a 'mid-surface' between the hanging surface and the floor surface. This surface, referred to as a track surface in tetrahedral modelling, is the path in three dimensions that the search ellipse follows, while maintaining the same ratio between the floor and hanging surface as a point selected from anywhere in the model.

4. Variography and Kriging With a Tetrahedral Model

4.1. Variography

In variography the distances and angles of sample pairs are calculated in the tetrahedral model space and not the Cartesian space. Following the successful generation of the tetrahedral model each sample point is located inside one tetrahedron. The coordinates in the tetrahedra are normalized so that the bottom surface has a Z of 0 and the top surface has a Z of 1. The space between the two surfaces has the original Cartesian coordinates and any number of other coordinate systems based on the tetrahedra. Different tetrahedral coordinate systems can be derived by starting at different places in the model. Neighbouring samples are found for each point, so that all pairs of points up to a radius of number of lags x lag size are found.

When passing from one tetrahedron to another, the incident angle from the old tetrahedron is converted to the coordinate system of the new tetrahedron. Keeping track of the apparent direction provides a bearing and distance. The Z coordinate (relative distance between the two surfaces) provides the tetrahedral Z coordinate. This process provides coordinates relative to the starting point. The end result is a search cone for building sample pairs that is distorted and follows the track surface of the tetrahedral model.

Once the sample pairs are constructed in the tetrahedral model space, experimental variograms are calculated as normal. The variogram lags and the azimuth and dip angles are referring now to the tetrahedral model space and not the original Cartesian space. All variogram models fitted and their parameters refer to the unfolded space. This must be considered when interpreting variography results produced using tetrahedral modelling.

4.2. Kriging

In grade estimation, the fundamental operation of unfolding is to list all samples inside an ellipsoid centred at a given coordinate. So, given a coordinate, the tetrahedron containing that coordinate is located. Samples from that tetrahedron are added, that are inside the search ellipsoid. Neighbouring tetrahedra are searched for more samples which are inside the search ellipsoid. This process is repeated until all relevant tetrahedra have been searched.

5. Conclusions

Tetrahedral modelling can be used in grade estimation and variography of deformed strata bound deposits, where mineralisation is controlled by a pair of structural surfaces. The search ellipse for variography and grade estimation is distorted from the usual regular ellipse to follow folded hanging and floor surfaces which represent the ore body geometry. The great benefit of using distorted search ellipses is that the block model stays in the position that it was created and the samples stay in their true position. Tools are provided to display the resultant tetrahedral model as a triangulation both in its whole form and as track surfaces for validation purposes. The results from the application of tetrahedral modelling in variography and grade estimation of a phosphate deposit confirmed the benefits of this approach.

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